

3.10/72

day 51

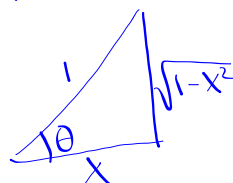
differentiate implicitly $\cos(\cos^{-1}(x)) = x$

$$-\sin(\cos^{-1}(x)) \cdot \frac{d}{dx}(\cos^{-1}(x)) = 1$$

$$\text{so } \frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sin(\cos^{-1}(x))} = \boxed{\frac{-1}{\sin(\theta)}}$$

but what is $\sin(\theta)$?
Recall: $\cos^{-1}(x)$ is "the angle whose cosine is x "
Let's call it θ

$$\text{so } \cos(\theta) = x$$



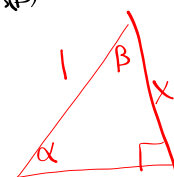
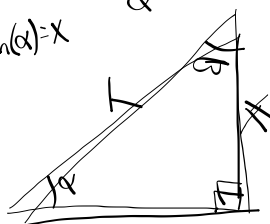
$$\text{so } \sin(\theta) = \sqrt{1-x^2}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

72b) Use $\underbrace{\sin^{-1}(x)}_{\alpha} + \underbrace{\cos^{-1}(x)}_{\beta} = \frac{\pi}{2}$

$$\sin(\alpha) = x$$

$$\cos(\beta) = x$$



diff
imp

$$\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$$

$$\frac{1}{\sqrt{1-x^2}} + \frac{d}{dx}(\cos^{-1}(x)) = 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

OOPS

$$\cos^{-1}(\cos(x)) = x$$

$$\frac{d}{dx}(\cos^{-1}(x)) \Big|_{x=\cos x} \cdot \frac{d}{dx}(\cos x) = 1$$

$$\frac{d}{dx}(\cos^{-1}(x)) \Big|_{x=\cos x} (-\sin x) = 1$$

$$\frac{d}{dx}(\cos^{-1}(x)) \Big|_{x=\cos x} = \frac{1}{-\sin(x)}$$

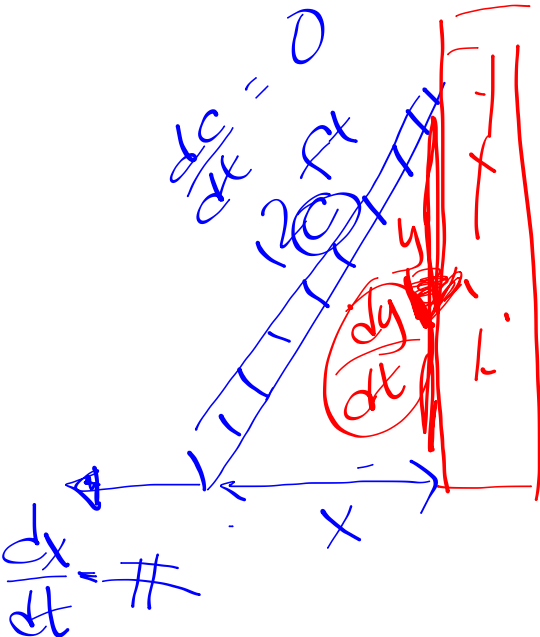
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$$\lim_{x \rightarrow 0} x \csc(x)$$
$$= x \frac{1}{\sin x}$$

Not $x \sin^{-1}(x)$ Nor $x \cos x$

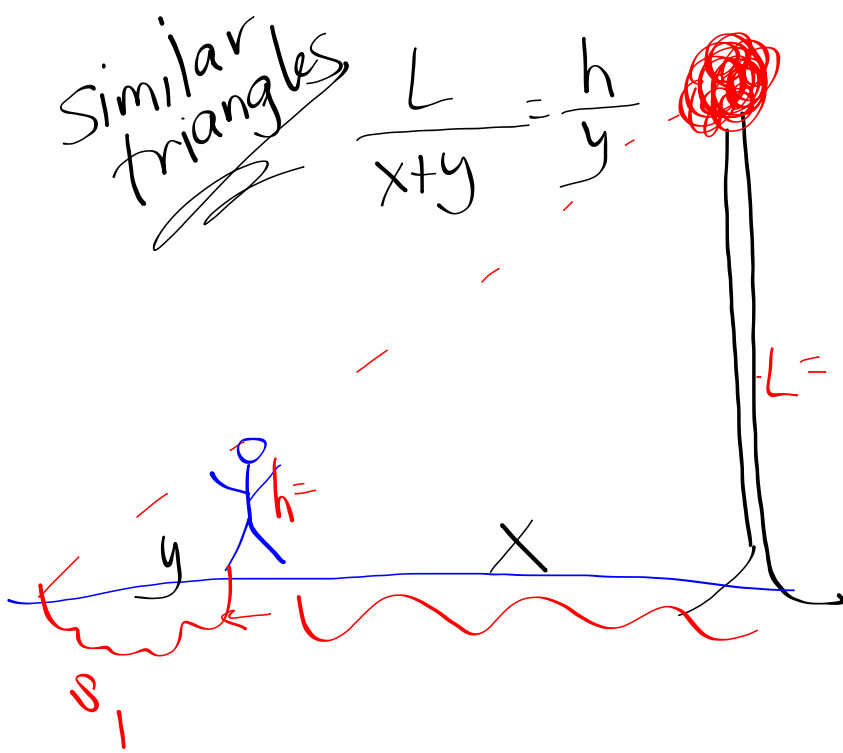
Nor ...

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$$x^2 + y^2 = 12^2$$

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Handwritten notes and diagrams illustrating a related rates problem.

Diagram 1 (Left): A green rhombus with a purple triangle inside. The triangle has a base of 20 ft and a height of 72 ft. The top vertex of the triangle is labeled "HSH".

Diagram 2 (Right): A black rhombus with a red triangle inside. The triangle has a base of 20 ft and a height of 72 ft. The top vertex of the triangle is labeled "day 51". The distance from the top vertex to the base is labeled d . The distance from the base to the right side is labeled $90 - x$. The right side is labeled 18 ft/sec .

Equations:

① $\frac{3 \cdot 11}{26}$

② $d^2 = y^2 + (90 - x)^2$

③ $2d \frac{dd}{dt} = 2y \frac{dy}{dt} + 2(90 - x) \left(-\frac{dx}{dt}\right)$

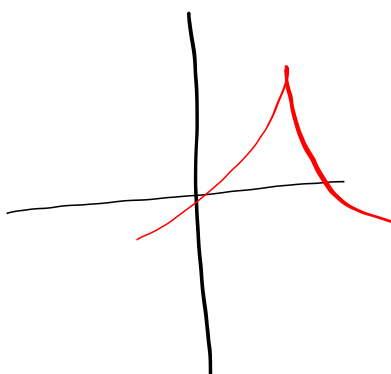
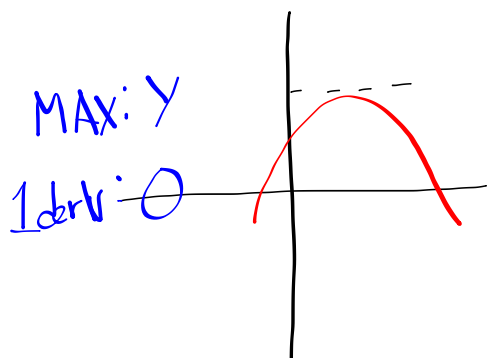
$\frac{d}{dt} \left(\frac{dd}{dt} \right) = y \frac{dy}{dt} - (90 - x) \frac{dx}{dt}$

$\sqrt{20^2 + 72^2} ?$

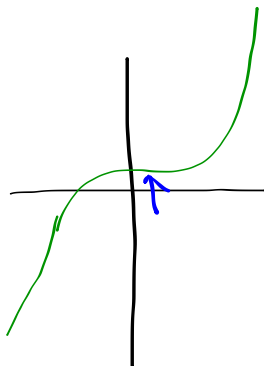
Recall in 4-1

Maxima/Minima

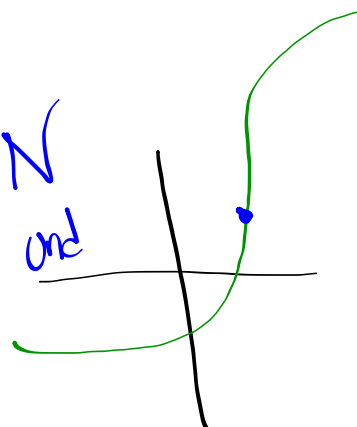
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MAX: N
1st deriv: 0



MAX: N
1st deriv: und



if $f'(a)=0$ or $f'(x)$ is undefined at $x=a$ day 51

then

extreme
values
MUST happen
at critical
points or
endpoints

$x=a$ is called a critical
and $f(x)$ may have an extreme
value there (max or min).

How do I know?

ABSOLUTE / GLOBAL MAX
↓
LOCAL / RELATIVE max

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